

## खोन्दो 8

# थामख 'नासुनि सिनायथिहोनाय ( Introduction to Trigonometry )

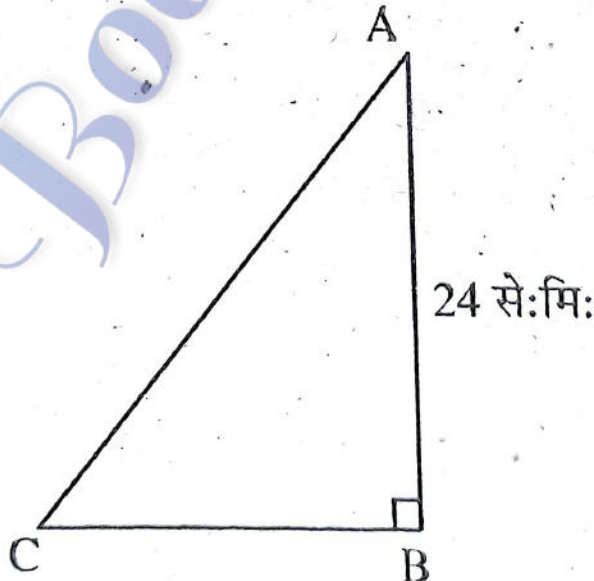
## सोनाये बिदां - 8.1

1.  $\Delta ABC$  आव, B आव ख 'नाथि,  $AB = 24 \text{ cm}$ ,  $BC = 7 \text{ cm}$  दिहुन -  
(i)  $\sin A$ ,  $\cos A$  (ii)  $\sin C$ ,  $\cos C$

सावफुंथाइ :- (i)  $\sin A$ ,  $\cos A$

$\Delta ABC$  आव  $\angle B = 90^\circ$ ,  $AB = 24$  से:मि:

$BC = 7$  से:मि:



$$\begin{aligned}
 \therefore AC^2 &= AB^2 + BC^2 \quad [\text{पाइथागोरसनि नेमै}] \\
 &= (24)^2 + 7^2 \\
 &= 576 + 49 \\
 &= 625
 \end{aligned}$$

$$\therefore AC = \sqrt{625} = 25 \text{ से:मि:}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{7}{25}$$

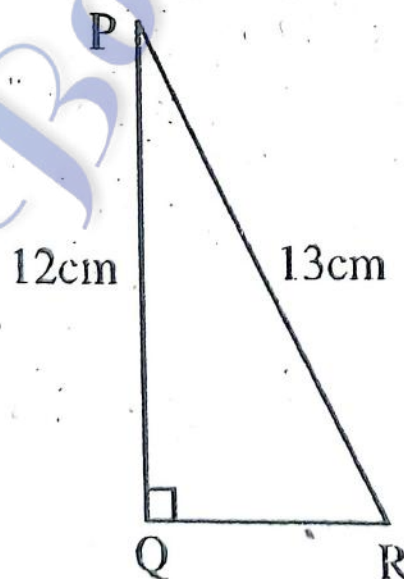
आरो  $\therefore \cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii)  $\sin C, \cos C$

$$\sin C = \frac{AB}{AC} = \frac{24}{25}$$

$$\cos C = \frac{BC}{AC} = \frac{7}{25}$$

2. सावगारि 8.13 आव, दिहुन  $\tan P - \cot R$ .



मावफुंथाइ :-  $\Delta PQR$  आव  $\angle Q = 90^\circ$

$$\therefore PR^2 = PQ^2 + QR^2 \quad [\text{पाइथागरासनि नेमै}]$$

$$\begin{aligned}\Rightarrow QR^2 &= PR^2 - PQ^2 \\ &= (13)^2 - (12)^2 \\ &= 169 - 144 \\ &= 25\end{aligned}$$

$$\therefore QR = \sqrt{25} = 5 \text{ से:मि:}$$

$$\therefore \tan P = \cot R$$

$$= \frac{QR}{PQ} = \frac{QR}{PQ} = 0$$

3. जुदि  $\sin A = \frac{3}{4}$ ,  $\cos A$  आरो  $\tan A$  खौ साननानै दिहुन।

मावफुंथाइ :- होनाय दड',  $\sin A = \frac{3}{4}$

जौ मिथिगौदि,

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \Rightarrow \cos^2 A &= 1 - \sin^2 A \\ &= 1 - \left(\frac{3}{4}\right)^2 \\ &= 1 - \frac{9}{16} \\ &= \frac{16 - 9}{16}\end{aligned}$$

$$= \frac{7}{16}$$

$$\therefore \cos A = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

आरोबाव,  $\cos A \sec A = 1$

$$\Rightarrow \sec A = \frac{1}{\cos A}$$

$$\Rightarrow \sqrt{1 + \tan^2 A} = \frac{1}{\cos A} \quad [\sec^2 A = 1 + \tan^2 A]$$

$$\Rightarrow 1 + \tan^2 A = \frac{1}{\cos^2 A} \quad [\text{फारनैथिंभो बर्ग खालामना}]$$

$$\Rightarrow \tan^2 A = \frac{1}{\cos^2 A} - 1$$

$$= \frac{1 - \left(\frac{\sqrt{7}}{4}\right)^2}{\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{1 - \frac{7}{16}}{\frac{7}{16}}$$

$$= \frac{16 - 7}{7} = \frac{9}{7}$$



$$= \frac{9}{16} \times \frac{16}{7}$$

$$= \frac{9}{7}$$

$$\therefore \tan A = \frac{3}{\sqrt{7}}$$

4. होनाय दं  $15 \cot A = 8$ ,  $\sin A$  आरो  $\sec A$  खौ दिहनु।

मावफुंथाइ :- होनाय दं,  $15 \cot A = 8$

$$\Rightarrow \cot A = \frac{8}{15}$$

जों मिथिगौदि,

$$\sin A \operatorname{cosec} A = 1$$

$$\Rightarrow \sin A = \frac{1}{\operatorname{cosec} A}$$

$$[\operatorname{cosec}^2 A = 1 + \cot^2 A]$$

$$= \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{8}{15}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{64}{225}}}$$

$$= \frac{1}{\sqrt{\frac{225 + 64}{225}}}$$

$$= \frac{1}{\sqrt{\frac{289}{225}}}$$

$$= \frac{1}{17}$$

$$15$$

$$= \frac{15}{17}$$

$$\therefore \sin A = \frac{15}{17}$$

आरोबाव,  $\sec^2 A = 1 + \tan^2 A$

$$\Rightarrow \sec A = \sqrt{1 + \tan^2 A}$$

$$= \sqrt{1 + \frac{1}{\cot^2 A}}$$

$$= \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}}$$

$$= \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

$$= \frac{\sqrt{\left(\frac{8}{15}\right)^2 + 1}}{\frac{8}{15}}$$

$$= \frac{\sqrt{\frac{64}{225} + 1}}{\frac{8}{15}}$$

$$= \frac{\sqrt{\frac{64 + 225}{225}}}{\frac{8}{15}}$$

$$= \frac{\sqrt{\frac{289}{225}}}{\frac{8}{15}}$$

$$= \frac{\frac{17}{15}}{\frac{8}{15}}$$

$$= \frac{17}{15} \times \frac{15}{8}$$

$$= \frac{17}{8}$$

$$\therefore \sec A = \frac{17}{8}$$

5. होनाय दं  $\sec \theta = \frac{13}{12}$ , गुबुन गासैबो थामखनांसुआरि रुजुथाइफोरखौ सानख 'नानै दिहुन।

मावफुंथाइ :- होनाय दड,  $\sec \theta = \frac{13}{12}$

$$\therefore \cos \theta = \frac{12}{13}$$

$$\therefore \sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$= \sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$= \sqrt{1 - \frac{144}{169}}$$

$$= \sqrt{\frac{169 - 144}{169}}$$

$$= \sqrt{\frac{5}{13}}$$

$$\therefore \operatorname{cosec} \theta = \frac{13}{5}$$

$$\begin{aligned} \therefore \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ &= \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12} \end{aligned}$$

$$\therefore \cot \theta = \frac{12}{5}$$

6. जुदि  $\angle A$  आरो  $\angle B$  ख'नासा ख'ना जाहाथे  $\cos A = \cos B$ , अब्ला

$$\angle A = \angle B$$

मावफुंथाइ :- हमना लाबाय,



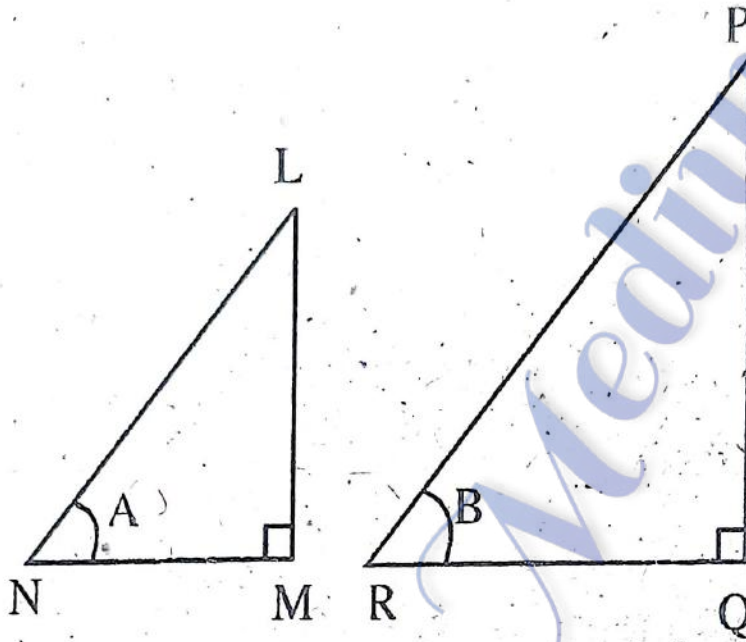
LMN आरो PQR आ मोननै खनाथि आखान्थिथाम  
जाहाथे,

$$\angle LNM = \angle A \text{ आरो } \angle PRQ = \angle B$$

होनाय दड,

$$\cos A = \cos B$$

$$\Rightarrow \frac{NM}{NL} = \frac{RQ}{RP}$$



$$\Rightarrow \frac{NM}{RQ} = \frac{NL}{RP} = K \quad (1)$$

जेराव K आ मोनसे दाजाबथाइ अनजिमा।

$$\Rightarrow NM = KRQ$$

$$NL = KRP$$

दानिया, पाइथागरासनि नेम बाहायना-

$$ML = \sqrt{NL^2 - NM^2}$$

$$= \sqrt{(KRP)^2 - (KRQ)^2}$$

$$= \sqrt{k^2 R^2 P^2 - K^2 R^2 Q^2}$$

$$= KR\sqrt{P^2 - Q^2}$$

$$\text{आरो } QP = \sqrt{(RP)^2 - (RQ)^2}$$

$$= R\sqrt{P^2 - Q^2}$$

$$\therefore \frac{ML}{QP} = \frac{KR\sqrt{P^2 - Q^2}}{R\sqrt{P^2 - Q^2}} = K \quad \text{--- (2)}$$

(1) आरो (2) ननिफ्राय

$$\frac{NM}{RQ} = \frac{NL}{RP} = \frac{ML}{QP}$$

$\therefore \triangle LMN \sim \triangle PQR$  [SSS महरसेआरि रादाइ]

$$\therefore \angle LNM = \angle PRQ$$

$$\Rightarrow \angle A = \angle B$$

दिन्थिनाय जाबाय।

7. जुदि  $\cot\theta = \frac{7}{8}$ , मानखौ दिहुन :

$$(i) \frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$(ii) \cot^2\theta$$

मावफुंथाइ :- (i) होनाय दड,

$$\cot\theta = \frac{7}{8}$$

$$\therefore \frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}$$

$$= \frac{1 - \frac{1}{\operatorname{cosec}^2 \theta}}{\sin^2 \theta} \quad \left[ \because \sin \theta = \frac{1}{\operatorname{cosec} \theta} \right]$$

$$= \frac{\operatorname{cosec}^2 \theta - 1}{\frac{1}{\operatorname{cosec}^2 \theta}}$$

$$= \frac{1 + \cot^2 \theta - 1}{\operatorname{cosec}^2 \theta} \times \frac{\operatorname{cosec}^2 \theta}{1}$$

$$= \left( \frac{7}{8} \right)^2$$

$$= \frac{49}{64}$$

$$(ii) = \cot^2 \theta = \left( \frac{7}{8} \right)^2 = \frac{49}{64}$$

8. જુદી  $3\cot A = 4$ , નાથગ્રોમ  $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$  નાંગૌના  
નડા।

માવફુંથાઈ : હોનાર્ય દડ,

$$3\cot A = 4$$

$$\cot A = \frac{4}{3}$$

∴ L.H.S.

$$\begin{aligned}& \frac{1 - \tan^2 A}{1 + \tan^2 A} \\&= \frac{1 - \frac{1}{\cot^2 \theta}}{1 + \frac{1}{\cot^2 \theta}} \quad [\cot A \cdot \tan A = 1] \\&= \frac{\cot^2 \theta - 1}{\cot^2 \theta + 1} \\&= \frac{\cot^2 \theta - 1}{\cot^2 \theta + 1} \\&= \frac{\left(\frac{4}{3}\right)^2 - 1}{\left(\frac{4}{3}\right)^2 + 1} \\&= \frac{\frac{16}{9} - 1}{\frac{16}{9} + 1} \\&= \frac{16 - 9}{16 + 9} \\&= \frac{7}{25}\end{aligned}$$



R.H.S.

$$\begin{aligned}& \cos^2 A - \sin^2 A \\&= 1 - \sin^2 A - \sin A \quad \left[ \because \sin^2 A + \cos^2 A = 1 \right] \\&= 1 - 2 \sin^2 A \\&= 1 - 2 \cdot \frac{1}{\operatorname{cosec}^2 A} \quad \left[ \sin A \cdot \operatorname{cosec} A = 1 \right] \\&= \frac{\operatorname{cosec}^2 A - 2}{\operatorname{cosec}^2 A} \\&= \frac{1 + \cot^2 A - 2}{1 + \cot^2 A} \\&= \frac{\left(\frac{4}{3}\right)^2 - 1}{1 + \left(\frac{4}{3}\right)^2} \\&= \frac{\frac{16}{9} - 1}{1 + \frac{16}{9}} \\&= \frac{\frac{16 - 9}{9}}{\frac{9 + 16}{9}} \\&= \frac{7}{25}\end{aligned}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

एबा  $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$

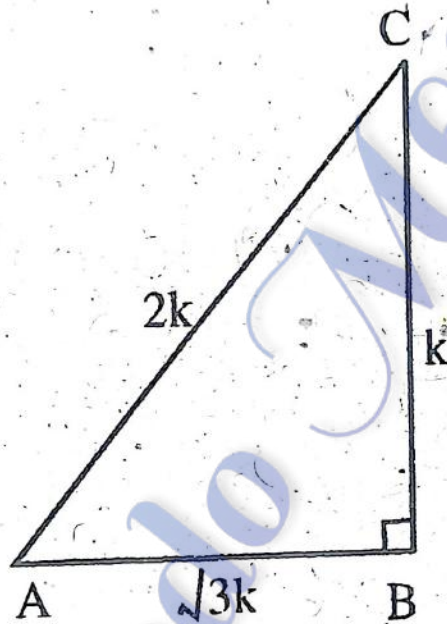
9. ABC आखान्थिथामआव, B आव ख'नाथि ख'ना, जुदि  $\tan A = \frac{1}{\sqrt{3}}$   
मानफोरखौ दिहुन :

(i)  $\sin A \cos C + \cos A \sin C$

(ii)  $\cos A \cos C - \sin A \sin C$

मावफुंथाइ :- हमना लाबाय,

ABC आ मोनसे खनाथि आखान्थिथाम आरो  $\angle B = 90^\circ$



होनाय दड',

$$\tan A = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{BC}{1} = \frac{AB}{\sqrt{3}} = K \quad (\text{हमबाय})$$

$$\therefore BC = K, AB = \sqrt{3} K$$

$\Delta ABC$  आव,  $\angle B = 90^\circ$

$$\therefore AC^2 = AB^2 + BC^2 \text{ [पाइथागोरसनि नेमै]}$$

$$= (\sqrt{3} K)^2 + K^2$$

$$= 3K^2 + K^2$$

$$= 4K^2$$

$$\therefore AC = \sqrt{4K^2} = 2K$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3} K}{2K} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3} K}{2K} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$$

दानिया, (i)  $\sin A \cos C + \cos A \sin C$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{1+3}{4} = 1$$

$$(ii) \cos A \cos C - \sin A \sin C$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

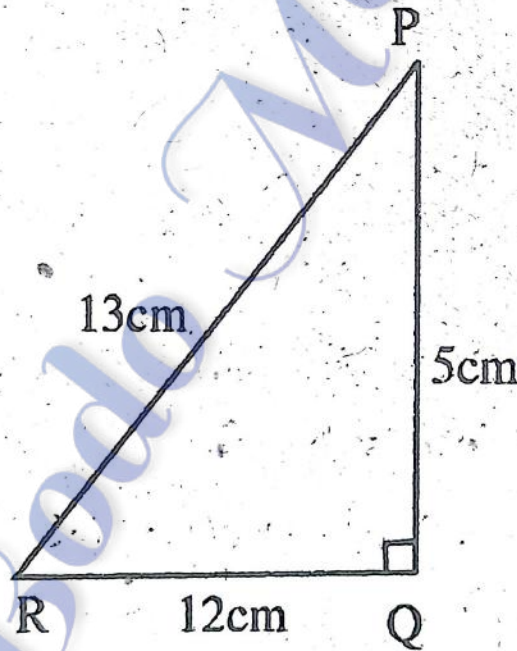
$$= 0$$

10.  $\Delta PQR$  આવ,  $Q$  આવ ધ 'નાથિલ' ના  $PR + QR = 25\text{cm}$  આરો  $PQ = 5\text{cm}$ .  $\sin P$ ,  $\cos P$  આરો  $\tan P$  નિ માનફોરલો દિહુન।

માલફુથાઈ :- હોનાય દડ.

$\Delta PQR$  આવ,  $\angle Q = 90^\circ$ ,  $PR + QR = 25$  સે:મિ:

$PQ = 5$  સે:મિ:



$$\therefore PR^2 = PQ^2 + QR^2 \quad [\text{પાઈથાગોરસનિ નેમ બાહાયના}]$$

$$\Rightarrow (25 - QR)^2 = 5^2 + QR^2$$

$$\Rightarrow 625 + QR^2 - 50QR = 25 + QR^2$$

$$\Rightarrow 50QR = 600$$

$$\Rightarrow QR = 12 \text{ સે.મિ.}$$



दानिया,  $PR + QR = 25$

$\Rightarrow PR = 25 - 12 \text{ से.मि.}$

$= 13 \text{ से.मि.}$

$\therefore \sin P = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

आरो  $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

11. गाहायाव होचाय बिबुंथिफोरा सैथो ना नंखाय बुं। नॉनि फिननायखौ थारजिर।

(i)  $\tan A$  नि मानआ जेब्लाबो 1 निखुइ दुइसिन।

(ii) A ख'नानि खायसे माननि थाखाय  $\sec A = \frac{12}{5}$

(iii) A ख'नानि Cosecant नि थाखाय बाहायनाय सुंथाब महरआ  $\cos A$ .

(iv)  $\cot A$  आ  $\cot A$  आरो A नि सानजाबगासै।

(v) माबेबा ख'ना '0' नि थाखाय  $\sin \theta = \frac{4}{3}$ ।

मावफुंथाइ :-

(i)  $\tan A = \frac{\text{थोंगोर}}{\text{हासा}}$ , आरो थोंगोरा हासानिखुइ लाउसिन जानो हागौ।  
 $\therefore$  गोरोन्थि।

(ii)  $\sec A = \frac{\text{लाउआखान्थि}}{\text{हासा}}$  आरो लाउखान्थिया हासानि  $\frac{12}{5}$  फान जानो हागौ।  
 $\therefore$  थार

(iii)  $\cos A$  આ  $\angle A$  નિ cosine નિ થાચાય સુંદ' ફોરમાયનાય।

$\therefore$  ગોરોન્થિ

(iv)  $\cot A$  આરો  $\angle A$  નિ Contansent નિ થાચાય બાહાયનાય સુંદ' ફોરમાયનાય।

$\therefore$  ગોરોન્થિ

(v) લાડઆચાન્થિયા ચનાથિ આચાન્થિથામનિ લાડસિન આચાન્થિ। બિનિચાયનો  $\sin A$  નિ માના જેબ્લાબો 1 નિચુઈ ચમ એબા સમાન।

$\therefore$  ગોરોન્થિ।

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## सौनाय बिदां - 8.2

1. गाहायाव होनायखौ मान दिहुन :

(i)  $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii)  $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii)  $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv)  $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 30^\circ + \cot 45^\circ}$

(v)  $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

(vi)  $\frac{\operatorname{cosec} 30^\circ + \operatorname{cosec} 60^\circ + \operatorname{cosec} 90^\circ}{\sec 0^\circ + \sec 30^\circ + \sec 60^\circ}$

मावफुंथाइ :-

(i)  $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$



$$(ii) \quad 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4}$$

$$= \frac{8+3-3}{4}$$

$$= \frac{8}{4}$$

$$= 2$$

$$(iii) \quad \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})}$$



$$= \frac{\sqrt{3} \times \sqrt{2} (1 - \sqrt{3})}{2\sqrt{2} (1 + \sqrt{3}) \times \sqrt{2} (1 - \sqrt{3})}$$

$$= \frac{\sqrt{6} - 3\sqrt{2}}{4(1 - 3)}$$

$$= \frac{3\sqrt{2} - \sqrt{6}}{8}$$

$$(iv) \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

$$= \frac{(3\sqrt{3} - 4)(3\sqrt{3} - 4)}{(3\sqrt{3} + 4)(3\sqrt{3} - 4)}$$

$$= \frac{(3\sqrt{3} - 4)^2}{9 \times 3 - 16}$$

$$= \frac{9 \times 3 + 16 - 24\sqrt{3}}{27 - 16}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

$$(v) \quad \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= 5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1 \quad \left[\because \sin^2 A + \cos^2 A = 1\right]$$

$$= 5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1$$

$$= \frac{5}{4} + \frac{16}{3} - 1$$

$$= \frac{15 + 64 - 12}{12}$$

$$= \frac{67}{12}$$

$$(vi) \quad \frac{\operatorname{cosec} 30^\circ + \operatorname{cosec} 60^\circ + \operatorname{cosec} 90^\circ}{\sec 0^\circ + \sec 30^\circ + \sec 60^\circ}$$

$$= \frac{\operatorname{cosec} 30^\circ + \operatorname{cosec} 60^\circ + \operatorname{cosec} 90^\circ}{\sec 0^\circ + \sec 30^\circ + \sec 60^\circ}$$

$$= \frac{2 + \frac{2}{\sqrt{3}} + 1}{1 + \frac{2}{\sqrt{3}} + 2}$$

$$\begin{aligned}
& \frac{2\sqrt{3} + 2 + \sqrt{3}}{\sqrt{3}} \\
&= \frac{\sqrt{3} + 2 + 2\sqrt{3}}{\sqrt{3}} \\
&= \frac{2\sqrt{3} + 2 + \sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3} + 2 + 2\sqrt{3}} \\
&= \frac{3\sqrt{3} + 2}{3\sqrt{3} + 2} \\
&= 1
\end{aligned}$$

2. गेबें थांखिखौ सायख' आरो नॉनि सायख' नायखौ थारजिर:

(i)  $\frac{2 \tan 30^\circ}{1 + \tan^2 45^\circ} =$

(A)  $\sin 60^\circ$

(B)  $\cos 60^\circ$

(C)  $\tan 60^\circ$

(D)  $\sin 30^\circ$

(ii)  $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

(A)  $\tan 90^\circ$

(B) 1

(C)  $\sin 45^\circ$

(D) 0

(iv)  $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

(A)  $\cos 60^\circ$

(B)  $\sin 60^\circ$

(C)  $\tan 60^\circ$

(D)  $\sin 30^\circ$

मावफुथाइ :-

$$(i) \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$= \sin 60^\circ$$

$\therefore \sin 60^\circ$  आ थार फिननाय।

$$(ii) \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - 1}{1 + 1} = 0$$

$\therefore (D) 0$  आ थार फिननाय।

$$(iii) \text{ जेब्ला, } A = 0^\circ$$

$$\sin 2A = \sin 2 \cdot (0^\circ) = \sin 0^\circ = 0$$



$$2 \sin A = 2 \sin 0^\circ = 2 \times 0 = 0$$

$\therefore$  (A)  $0^\circ$  आ थार फिननाय।

$$(iv) \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$= \sqrt{3}$$

$$= \tan 60^\circ$$

$\therefore$  (C)  $\tan 60^\circ$  आ थार फिननाय।

$$3. (i) \text{ जुदि } \tan (A+B) = \sqrt{3} \text{ आरो } \tan (A-B) = \frac{1}{\sqrt{3}};$$

$0^\circ < A+B \leq 90^\circ$ ;  $A > B$ , A आरो B खौ दिहनु।

मावफुंथाइ :- होनाय दङ्

$$\tan (A+B)=\sqrt{3}$$

$$\Rightarrow \tan (A+B)=\tan 60^{\circ}$$

$$\Rightarrow A+B=60^{\circ} \text{ ————— (1)}$$

आरो,  $\tan (A-B)=\frac{1}{\sqrt{3}}$

$$\Rightarrow \tan (A-B)=\tan 30^{\circ}$$

$$\Rightarrow A-B=30^{\circ} \text{ ————— (2)}$$

$$(1)+(2) \Rightarrow 2A=90^{\circ}$$

$$\Rightarrow A=45^{\circ}$$

(1) नंआव  $A=45^{\circ}$  खौ फजना

$$45^{\circ}+B=60^{\circ}$$

$$\Rightarrow B=15^{\circ}$$

$\therefore$  दि: गो:  $A=45^{\circ}$ ,  $B=15^{\circ}$

3. (ii) जिदु  $\sin(x+y)=1$ ,  $\cos(x-y)=\frac{\sqrt{3}}{2}$  आरो  $x>y$ ,

$0^{\circ} \leq x+y \leq 90^{\circ}$  अब्ला  $x$  आरो  $y$  दिहुन।

मावफुंथाइ:  $\sin(x+y)=1$

$$\Rightarrow \sin (x+y)=\sin 90^{\circ}$$

$$\Rightarrow x+y=90^{\circ}$$

$$\Rightarrow x=90^{\circ}-y \text{ .....(i)}$$

आरोबाव,  $\cos(x-y)=\frac{\sqrt{3}}{2}$

$$\Rightarrow \cos(x - y) = \cos 30^\circ$$

$$\Rightarrow \cos(x - y) = \cos 30^\circ$$

$$\Rightarrow x - y = 30^\circ$$

$$\Rightarrow 90^\circ - y - y = 30^\circ \text{ ((i)नं निफ्राय)}$$

$$\Rightarrow 90^\circ - 2y = 30^\circ$$

$$\Rightarrow -2y = 30^\circ - 90^\circ$$

$$\Rightarrow -2y = -60^\circ$$

$$\Rightarrow y = \frac{-60^\circ}{-2} = 30^\circ$$

$\therefore y$  नि मानखौ (i)आव फजना-

$$\begin{aligned} x &= 90^\circ - y \\ &= 90^\circ - 30^\circ \end{aligned}$$

$$\therefore x = 60^\circ$$

$$y = 30^\circ$$

4. गाहायाव होनाय बिबंथिफोरा गेबें ना गोरोन्थि बुं। नॉनि फिननायखौ थारजिर।

(i)  $\sin(A + B) = \sin A + \sin B$ .

(ii)  $\theta$  या देरलाडेब्ला  $\sin \theta$  नि मानआ देरलाडे।

(iii)  $\theta$  या देरलाडेब्ला  $\cos \theta$  नि मानआ देरलाडे।

(iv)  $\theta$  नि गासैबो माननि थाखाय  $\sin \theta = \cos \theta$

(v)  $A = 0^\circ$  नि थाखाय  $\cot A$  आ बुंफोरथाइ गैया।

मावफुंथाइ :- हमना लाबाय,

(i)  $A = 30^\circ, B = 30^\circ$



$$\therefore \sin (A + B) = \sin (30^0 + 30^0) = \sin 60^0 = \frac{\sqrt{3}}{2}$$

आरो  $\sin A + \sin B = \sin 30^0 + \sin 30^0 = \frac{1}{2} + \frac{1}{2} = 1$

$$\therefore \sin (A + B) \neq \sin A + \sin B$$

$\therefore$  गोरान्थि।

(ii)  $\theta$  नि माना बारायोब्ला  $\sin \theta$  नि मानाबो बारायगोन।

$\therefore$  थार।

(iii)  $\theta$  नि माना बारायोब्ला  $\cos \theta$  नि मानाबो बारायगोन।

$\therefore$  गोरान्थि।

(iv) हमना लाबाय,  $\theta = 60^0$

$$\therefore \sin \theta = \sin 60^0 = \frac{\sqrt{3}}{2}$$

आरो  $\cos \theta = \cos 60^0 = \frac{1}{2}$

$$\therefore \sin \theta \neq \cos \theta \quad [\theta \text{ नि जायखि जाया माननि थाखाय}]$$

$\therefore$  गोरान्थि।

(v) मानफोरनि फारिलाइ बादियै,

$\therefore$  थार।

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### सौनाय बिदां - 8.3

1. मान दिहनु :

(i)  $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii)  $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii)  $\cos 48^\circ - \sin 42^\circ$

(iv)  $\operatorname{cosec} 31^\circ - \sec 59^\circ$

(v)  $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$

(vi)  $\tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 30^\circ$

(vii)  $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$

(viii)  $3 \frac{\sin 23^\circ}{\cos 67^\circ} + 4 \frac{\sec 47^\circ}{\operatorname{cosec} 43^\circ}$

(ix)  $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ$

मावफुंथाइ :-

(i)  $\frac{\sin 18^\circ}{\cos 72^\circ}$

$$= \frac{\sin 18^\circ}{\cos (90^\circ - 18^\circ)}$$

$$= \frac{\sin 18^\circ}{\sin 18^\circ} \quad [\because \cos (90^\circ - \theta) = \sin \theta]$$

$$= 1$$

$$(ii) \quad \frac{\tan 26^\circ}{\cot 64^\circ}$$

$$= \frac{\tan 26^\circ}{\cot (90^\circ - 26^\circ)}$$

$$= \frac{\tan 26^\circ}{\tan 26^\circ} \quad [\because \cot(90^\circ - \theta) = \tan \theta]$$

$$= 1$$

$$(iii) \quad \cos 48^\circ - \sin 42^\circ$$

$$= \cos (90^\circ - 42^\circ) - \sin 42^\circ$$

$$= \sin 42^\circ - \sin 42^\circ \quad [\because \cos(90^\circ - \theta) = \sin \theta]$$

$$= 0$$

$$(iv) \quad \operatorname{cosec} 31^\circ - \sec 59^\circ$$

$$= \operatorname{cosec} 31^\circ - \sec (90^\circ - 31^\circ)$$

$$= \operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ \quad [\because \sec(90^\circ - \theta) = \operatorname{cosec} \theta]$$

$$= 0$$

$$v) \quad \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$$

$$= \sin (90^\circ - 55^\circ) \sin (90^\circ - 35^\circ) - \cos 35^\circ \cos 55^\circ$$

$$= \cos 55^\circ \cos 35^\circ - \cos 35^\circ \cos 55^\circ$$

$$= 0$$

$$\therefore \text{दि: गो: मान} = 0$$

$$(vi) \quad \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 30^\circ$$

$$= \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 30^\circ$$

$$= \tan (90^\circ - 55^\circ) \times \sqrt{3} \tan 55^\circ \times \frac{1}{\sqrt{3}}$$

$$= \cot 55^\circ \tan 55^\circ$$

$$= \frac{1}{\tan 55^\circ} \times \tan 55^\circ$$

$$= 1$$

∴ दि: गो: मान = 1

$$(vii) \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$

मावफुंथाइ :

$$\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$

$$= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2$$

$$= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2$$

$$= 1 + 1 - 2$$

$$= 2 - 2$$

$$= 0$$

∴ दि:गो: मान = 0

$$(viii) 3 \frac{\sin 23^\circ}{\cos 67^\circ} + 4 \frac{\sec 47^\circ}{\operatorname{cosec} 43^\circ}$$

मावफुंथाइ:

$$3 \frac{\sin 23^\circ}{\cos 67^\circ} + 4 \frac{\sec 47^\circ}{\operatorname{cosec} 43^\circ}$$



$$= \frac{3 \sin(90^\circ - 67^\circ)}{\cos 67^\circ} + \frac{4 \sec(90^\circ - 43^\circ)}{\operatorname{cosec} 43^\circ}$$

$$= \frac{3 \cos 67^\circ}{\cos 67^\circ} + \frac{4 \sec 43^\circ}{\sec 43^\circ}$$

$$= 3 + 4$$

$$= 7$$

∴ दि: गो: मान = 7

(ix)  $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ$

मावफुंथाइ :  $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ$

$$= \tan (90^\circ - 85^\circ) \tan (90^\circ - 65^\circ) \times \frac{1}{\sqrt{3}} \times \tan 65^\circ \tan 85^\circ$$

$$= \cot 85^\circ \cdot \cot 65^\circ \times \frac{1}{\sqrt{3}} \times \frac{1}{\cot 65^\circ} \times \frac{1}{\cot 85^\circ}$$

$$= \frac{1}{\sqrt{3}}$$

∴ दि: गो: मान =  $\frac{1}{\sqrt{3}}$

2. दिन्थि दि :

(i)  $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii)  $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

मावफुंथाइ :-

(i)  $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$$= \tan (90^\circ - 42^\circ) \tan 23^\circ \tan 42^\circ \tan (90^\circ - 23^\circ)$$

$$= \cot 42^\circ \tan 23^\circ \tan 42^\circ \cot 23^\circ$$



$$= \frac{1}{\tan 42^\circ} \times \tan 23^\circ \tan 42^\circ \times \frac{1}{\tan 23^\circ}$$

$$= 1$$

$$\therefore \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$$

दिन्थिनाय जाबाय।

$$(ii) \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

$$= \cos 38^\circ \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin (90^\circ - 38^\circ)$$

$$= \cos 38^\circ \sin 38^\circ - \sin 38^\circ \cos 38^\circ$$

$$= 0$$

$$\therefore \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$$

दिन्थिनाय जाबाय।

3. जुदि  $\tan 2A = \cot (A - 18^\circ)$ , जेराव  $2A$  मोनसे ख'नासा ख'ना,  $A$  नि मानखौ दिहुन।

मावफुंथाइ :- होनाय दड',

$$\tan 2A = \cot (A - 18^\circ)$$

$$\Rightarrow \cot (90^\circ - 2A) = \cot (A - 18^\circ) \quad [\because \cot (90^\circ - \theta) = \tan \theta]$$

$$\Rightarrow 90^\circ - 2A = A - 18^\circ$$

$$\Rightarrow 3A = 108^\circ$$

$$\Rightarrow A = 36^\circ$$

$\therefore$  दि: गो:  $A$  नि माना  $36^\circ$

4. जुदि  $\tan A = \cot B$ , फोरमान खालाम दि  $A + B = 90^\circ$ ,

मावफुंथाइ :- होनाय दड',

$$\tan A = \cot B$$

$$\Rightarrow \tan A = \tan (90^\circ - B) \quad [\because \tan (90^\circ - \theta) = \cot \theta]$$

$$\Rightarrow A = 90^\circ - B$$

$$\Rightarrow A + B = 90^\circ$$

फोरमान जाबाय।

5. जुदि  $\sec 4A = \operatorname{cosec} (A - 20^\circ)$ , जेराव  $4A$  आ मोनसे ख'ना,  $A$  नि मानखौ दिहुन।

मावफुंथाइ :- होनाय दङ्,

$$\sec 4A = \operatorname{cosec} (A - 20^\circ)$$

$$\Rightarrow \operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 20^\circ)$$

$$[\because \operatorname{cosec} (90^\circ - \theta) = \sec \theta]$$

$$\Rightarrow 90^\circ - 4A = A - 20^\circ$$

$$\Rightarrow 5A = 110$$

$$\Rightarrow A = 22^\circ$$

$\therefore$  दि: गो:  $A$  नि माना  $22^\circ$ ।

6. जुदि  $A, B$  आरो  $C$  आ मोनसे आखान्थिथाम  $\triangle ABC$  नि इसिं ख'ना जायो,

$$\text{अब्ला दिन्थि दि } \sin \left( \frac{B+C}{2} \right) = \cos \frac{A}{2}$$

मावफुंथाइ :-

$$\text{L.H.S.} \quad \sin \left( \frac{B+C}{2} \right)$$

$$= \sin \left( \frac{180^\circ - A}{2} \right) \quad [\because A + B + C = 180^\circ]$$

$$= \sin \left( 90^\circ - \frac{A}{2} \right)$$

$$= \cos \frac{A}{2} \quad [\therefore \sin(90^\circ - \theta) = \cos \theta]$$

$$\therefore \sin \left( \frac{B+C}{2} \right) = \cos \frac{A}{2}$$

दिन्थिनाय जाबाय।

7.  $\sin 67^\circ + \cos 75^\circ$  खौ  $0^\circ$  आरो  $45^\circ$  ख'नानि गोजेराव थानाय  
थामख'नासुआरि रुजुथाइफोरनि महरआव फोरमाय।  
मावफुंथाइ :- होनाय दड'

$$\begin{aligned} & \sin 67^\circ + \cos 75^\circ \\ &= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ) \\ &= \cos 23^\circ + \sin 15^\circ \end{aligned} \quad \left| \begin{array}{l} \therefore \sin(90^\circ - \theta) = \cos \theta \\ \cos(90^\circ - \theta) = \sin \theta \end{array} \right.$$

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### सौनाय बिदां - 8.4

1. थामख 'नासु रुजुथाइ  $\sin A$ ,  $\sec A$  आरो  $\tan A$  खौ  $\cot A$  नि महरआव फोरमाय।

मावफुंथाइ :-  $\sin A = \frac{1}{\operatorname{cosec} A}$

$$= \sqrt{\frac{1}{1 + \cot^2 A}} \quad \text{--- (1) } [\operatorname{cosec}^2 A = 1 + \cot^2 A]$$

$$\sec A = \sqrt{1 + \tan^2 A} \quad [\sec^2 A = 1 + \tan^2 A]$$

$$= \sqrt{1 + \frac{1}{\cot^2 A}}$$

$$= \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}}$$

$$= \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}} \quad \text{--- (2)}$$

$$\tan A = \frac{1}{\cot A} \quad \text{--- (3)}$$

2.  $\angle A$  नि गुबुन गासैबो थामख 'नासुआरि रुजुथाइ फोरखौ  $\sec A$  नि महरआव फोरमाय।



मावफुंथाइ :-  $\cos A = \frac{1}{\sec A}$  — (1)

$\tan A = \sqrt{\sec^2 A - 1}$  — (2)

$$\left[ \begin{array}{l} \tan^2 A + 1 = \sec^2 A \\ \therefore \tan A = \sqrt{\sec^2 A - 1} \end{array} \right]$$

$\cot A = \frac{1}{\tan A}$   
 $= \frac{1}{\sqrt{\sec^2 A - 1}}$  — (3)

$\operatorname{cosec} A = \sqrt{1 + \cot^2 A}$  [ $\operatorname{cosec}^2 A = 1 + \cot^2 A$ ]

$$\begin{aligned} &= \sqrt{1 + \frac{1}{\sec^2 A - 1}} \\ &= \sqrt{\frac{\sec^2 A}{\sec^2 A - 1}} \\ &= \frac{\sec A}{\sqrt{\sec^2 A - 1}} \quad \text{— (4)} \end{aligned}$$

$\sin A = \frac{1}{\operatorname{cosec} A}$   
 $= \frac{1}{\frac{\sec A}{\sqrt{\sec^2 A - 1}}}$   
 $= \frac{\sqrt{\sec^2 A - 1}}{\sec A}$  — (5)

3. मान दिहुन :

(i)  $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii)  $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

मावफुंथाइ :-

(i)  $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{\sin^2 (90^\circ - 27^\circ) + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 (90^\circ - 17^\circ)}$$

$$= \frac{\sin^2 27^\circ + \cos^2 27^\circ}{\sin^2 17^\circ + \cos^2 17^\circ}$$

$$[\sin(90^\circ - \theta) = \cos \theta]$$

$$= \frac{1}{1}$$

$$[\cos(90^\circ - \theta) = \sin \theta]$$

$$= 1$$

(ii)  $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$$= \sin 25^\circ \cos (90^\circ - 25^\circ) + \cos 25^\circ \sin (90^\circ - 25^\circ)$$

$$= \sin 25^\circ \sin 25^\circ + \cos 25^\circ \cos 25^\circ$$

$$= \sin^2 25^\circ + \cos^2 25^\circ \quad [\sin^2 \theta + \cos^2 \theta = 1]$$

$$= 1$$

4. गेबें थांखिखौ सायख'। नॉनि सायख 'नायखौ थारजिर।

(i)  $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii)  $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) =$

(A) 0

(B) 1

(C) 2

(D) -1

(iii)  $(\sec A + \tan A)(1 - \sin A) =$

(A)  $\sec A$

(B)  $\sin A$

(C)  $\operatorname{cosec} A$

(D)  $\cos A$



$$(iv) \frac{1 + \tan^2 A}{1 + \cot^2 A} =$$

- (A)  $\sec^2 A$       (B)  $-1$       (C)  $\cot^2 A$   
 (D)  $\tan^2 A$

सावफुंथाइ :-

$$\begin{aligned} (i) \quad & 9 \sec^2 A - 9 \tan^2 A \\ &= 9 (\sec^2 A - \tan^2 A) \\ &= 9 \times 1 \\ &= 9 \end{aligned}$$

$\therefore$  (B) 9 आ थार फिननाय।

$$\begin{aligned} (ii) \quad & (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) \\ &= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \\ &= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \\ &= \frac{(\sin \theta + \cos \theta + 1)(\sin \theta + \cos \theta - 1)}{\sin \theta \cos \theta} \\ &= \frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta \cos \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\ &= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\ &= 2 \end{aligned}$$

∴ (C) 2 आ थार फिननाय।

(iii)  $(\sec A + \tan A) (1 - \sin A)$

$$= \left( \frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A)$$

$$= \left( \frac{1 + \sin A}{\cos A} \right) (1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

∴ (D)  $\cos A$  आ थार फिननाय।

(iv)  $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$$= \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}}$$

$$= \frac{1 + \tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}}$$

$$= (1 + \tan^2 A) \times \frac{\tan^2 A}{(1 + \tan^2 A)}$$

$$= \tan^2 A$$

∴ (D)  $\tan^2 A$  आ थार फिननाय।



5. ગાહાયનિ થાબિથાફોરખૌ ફોરમાન ખાલામ, જેરાવ લોબબા થાનાય  
 ખ 'નાફોરા જાયનિ થાચાય ફોરમાયથિફોરખૌ બુંફોરથાઇ હોનાય જાદોં  
 બેસોરો ખ 'નાસા ખ 'ના ।

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

[ ઇસારા : ફોરમાયથિફોરખૌ  $\sin \theta$  આરો  $\cos \theta$  નિ મહર આવ લિર ]

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

[ L.H.S. આરો R.H.S. યૌ જુદા જુદાયૈ જેથો સિફાય ]

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} =$$

$$\operatorname{cosec} A + \cot A, \operatorname{cosec}^2 A = 1 + \cot^2 A.$$

થાબિથાખૌ બાહાયનાનૈ

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A - \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$(ix) \quad (\operatorname{cosec} A - \sin A) (\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

[ इसारा : L.H.S. आरो R.H.S. खौ जुदा जुदायै जेथो सिफाय ]

$$(x) \quad \left( \frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left( \frac{1 - \tan A}{1 - \cot A} \right) = \tan^2 A$$

सावफुंथाइ :-

$$(i) \quad (\operatorname{cosec} \theta - \cot^2 \theta) = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\begin{aligned} \text{L.H.S.} \quad & (\operatorname{cosec} \theta - \cot \theta)^2 \\ &= \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\ &= \left( \frac{1 - \cos \theta}{\sin \theta} \right)^2 \\ &= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\ &= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} \\ &= \frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\ &= \frac{1 - \cos \theta}{1 + \cos \theta} \\ &= \text{R.H.S.} \end{aligned}$$



$$(ii) \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$\text{L.H.S.} \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{\cos A (1 + \sin A)}$$

$$= \frac{1 - \sin^2 A + 1 + \sin^2 A + 2 \sin A}{\cos A (1 + \sin A)}$$

$$= \frac{2 + 2 \sin A}{\cos A (1 + \sin A)}$$

$$= \frac{2(1 + \sin A)}{\cos A (1 + \sin A)}$$

$$= 2 \cdot \frac{1}{\cos A}$$

$$= 2 \sec A$$

$$= \text{R.H.S.}$$

$$(iii) \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\text{L.H.S.} \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}}$$



$$\begin{aligned}
&= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
&= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)} \\
&= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \\
&= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} \\
&= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} \\
&= \frac{1 + \sin \theta \cos \theta}{\sin \theta \cos \theta} \\
&= \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} \\
&= \frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta} + 1 \\
&= \sec \theta \operatorname{cosec} \theta + 1 \\
&= 1 + \sec \theta \operatorname{cosec} \theta \\
&= \text{R.H.S.}
\end{aligned}$$

$$(iv) \quad \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\text{L.H.S.} \quad \frac{1 + \sec A}{\sec A}$$

$$= \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}}$$

$$= 1 + \cos A$$

$$= \frac{(1 + \cos A)(1 - \cos A)}{1 - \cos A}$$

$$= \frac{1 - \cos^2 A}{1 - \cos A}$$

$$= \frac{\sin^2 A}{1 - \cos A} \quad \left[ \because \sin^2 A + \cos^2 A = 1 \right]$$

$$= \text{R.H.S.}$$

$$(v) \quad \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

$$\text{L.H.S.} \quad \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$



$$\begin{aligned}
&= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} \\
&= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\
&= \frac{\{(\cot A + \operatorname{cosec} A) - 1\} \{(\cot A + \operatorname{cosec} A)\}}{\{(\cot A - \operatorname{cosec} A) + 1\} \{(\cot A + \operatorname{cosec} A)\}} \\
&= \frac{\{(\cot A + \operatorname{cosec} A) - 1\} (\cot A + \operatorname{cosec} A)}{\{(\cot A + \operatorname{cosec} A)(\cot A - \operatorname{cosec} A) + (\cot A + \operatorname{cosec} A)\}} \\
&= \frac{\{(\cot A + \operatorname{cosec} A) - 1\} (\cot A + \operatorname{cosec} A)}{(\cot^2 A - \operatorname{cosec}^2 A) + (\cot A + \operatorname{cosec} A)} \\
&= \frac{\{(\cot A + \operatorname{cosec} A) - 1\} (\cot A + \operatorname{cosec} A)}{\{(\cot A + \operatorname{cosec} A) - 1\}} \\
&= \cot A + \operatorname{cosec} A \\
&= \text{R.H.S.}
\end{aligned}$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$\text{L.H.S.} \sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$



$$= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \text{R.H.S.}$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$\text{L.H.S.} \quad \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\sin \theta (\sin^2 \theta + \cos^2 \theta - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - \sin^2 \theta - \cos^2 \theta)}$$

$$[\sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{\sin \theta (\cos^2 \theta - \sin \theta)}{\cos \theta (\cos^2 \theta - \sin^2 \theta)}$$

$$= \tan \theta$$

$$= \text{R.H.S.}$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

L.H.S.

$$\begin{aligned} & (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\ &= (\sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A) \\ & \quad + (\cos^2 A + \sec^2 A + 2 \cos A \sec A) \\ &= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A \\ & \quad + 2 \sin A \operatorname{cosec} A + 2 \cos A \sec A \\ &= 1 + 1 + \cot^2 A + 1 + \tan^2 A + \\ & \quad 2 \sin A \cdot \frac{1}{\sin A} + 2 \cos A \cdot \frac{1}{\cos A} \\ &= 7 + \tan^2 A + \cot^2 A \\ &= \text{R.H.S.} \end{aligned}$$

$$(ix) (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

L.H.S.  $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$\begin{aligned} &= \left( \frac{1}{\sin A} - \sin A \right) \left( \frac{1}{\cos A} - \cos A \right) \\ &= \left( \frac{1 - \sin^2 A}{\sin A} \right) \left( \frac{1 - \cos^2 A}{\cos A} \right) \\ &= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \\ &= \sin A \cos A \\ &= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} \quad [\because \sin^2 A + \cos^2 A = 1] \\ &= \frac{\sin A \cos A}{\sin A \cos A} \\ &= \frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A} \end{aligned}$$



$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A}$$

$$= \text{R.H.S.}$$

$$(x) \quad \frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A$$

$$\text{L.H.S.} \quad \frac{1 + \tan^2 A}{1 + \cot^2 A}$$

$$= \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}}$$

$$= \frac{1 + \tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}}$$

$$= 1 + \tan^2 A \times \frac{\tan^2 A}{1 + \tan^2 A}$$

$$= \tan^2 A$$

$$= \text{R.H.S.}$$

$$\text{आरोबाव,} \quad \left( \frac{1 - \tan A}{1 - \cot A} \right)^2$$

$$= \left( \frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2$$



$$= \left( \frac{1 - \tan A}{\tan A - 1} \right)^2$$

$$= \left( 1 - \tan A \times \frac{(\tan A)}{1 - \tan A} \right)^2$$

$$= (-\tan A)^2$$

$$= \tan^2 A$$

$$= \text{R.H.S.}$$

$$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \left( \frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

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